

OPTIMUM COST DESIGN OF R. C. ONE WAY SLABS

Rabi' M. Najem

Assistant Lecturer
Mosul University

Abstract

In this research , the formulation of optimum cost design for one way reinforced concrete slabs is presented , since it is useful and have a widespread usage among practicing engineering and applied to realistic structures subjected to the actual constraints of commonly used design codes such as the American concrete instituted code (ACI,2005).

The formulation contains minimizing an objective function that represent the cost of steel reinforcement and the cost of concrete , which is subjected to many constraints containing : flexural constraints , serviceability constraints and deflection constraints that illustrated with details in this paper . the optimum solution is calculated using the lagrangian multipliers method , and a visual basic computer software were developed to find the optimum solution .

Key Words: Optimization , Lagrangian Multiplier , Optimum Cost Design . One way Slabs .

التصميم الامثل للكلفة للبلاطات المسلحة أحادية الاتجاه

ربيع مؤيد نجم

مدرس مساعد

جامعة الموصل

الخلاصة

في هذا البحث تم استنباط صيغة لاجاد التصميم الامثل من ناحية الكلفة للبلاطات المسلحة احادية الاتجاه نظراً لفائدتها وأستخدامها الواسع في مجال الهندسة المدنية التطبيقية وأمكانية تطبيقها على منشآت واقعية خاضعة لأحمال ومحددات تصميمية من المدونات المعتمدة في التصميم للهندسة المدنية كمدونة الـ ACI , 2005 .

تتمثل صياغة هذه الطريقة بتطبيق الامثلية على دالة الهدف التي تمثل كلفة حديد التسليح وكلفة الخرسانة ، والتي تكون خاضعة لمحددات تصميمية مثل : محددات الانحناء ، محددات الديمومة ومحددات الهبوط والموضحة بالتفصيل في هذا البحث . تم أستخدام طريقة لاكرانج لحساب الحل الامثل .

Introduction

The goal of optimization is to find the values of the variables in the process that yield the best value of the performance criterion . A trade – off usually exist between capital and operating costs . The described factors – process or model of the performance criterion – constitute the optimization problem .

Engineers work to improve the initial design of equipment and strive to enhance the operation of that equipment once it is installed so as to realize the largest production , the greatest profit , the minimum cost , the least energy usage , and so on . Monetary value provides a convenient measure of different but otherwise incompatible objectives , but not all problems have to be considered in a monetary (cost versus revenue) framework [1] .

The optimum designs can be obtained by any one of the following methods :

- 1 – Experimental methods , in this method , designers conduct experimental studies on a finite number of models with different parameters , with the help of charts and tables prepared from experimental studies , the designers selects the best design .
- 2 – Classical methods , these are the analytical methods and they make use of differential calculus for locating the optimum design points . Necessary and sufficient conditions for a point to be a maximum or minimum can be derived even for multivariable problems .
- 3 – Computer aided design , in this method , the designer write program to analyse , design and estimate the cost of the structure , the results where studied after trying several designs , and the best one was selected as optimum .
- 4 – Automated optimum design , the designer writes program for analysis and design , Then he identifies the criteria for selecting the best design . On the available design a suitable optimization technique is employed to reach the optimum design point .

The standard form of a mathematical programming problem is

$$\text{Find } X = \begin{Bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_n \end{Bmatrix}$$

Which minimize $Z = f (x)$

Subject to $g_i (x) < 0$, where $i = 1 , 2 , \dots , n$.

n : no. of design variables .

In the above statement X is a vector of n design variables , $Z = f (x)$ is the objective function and g_i are design constraints .

Objective function is defined as a function of the design variables , the value of which provides the basis of choice between alternate acceptable designs . The objective may be minimization of weight , cost or stress concentration factor or it may be maximization of efficiency . In structural design the objective function is usually weight or cost minimization .

The constraints of a problem are the restriction which are to be satisfied to make a design acceptable , and in structural design may be classified into : 1- behavior constraints which are related to design variables implicitly and usually structural analysis is necessary to

evaluate them like the limitation on stresses , displacement and stability requirements . 2-Side constraints (Geometric constraints) which termed as a specified limitations in explicit form on a design variable or a relationship among a group of variables like the codal provisions of minimum or maximum values on a design variable , in other words , they limit the range of acceptable designs in the problem [2] .

Some of structural optimization researches deal with minimization of the weight of a structure , and others deal with minimizing the cost of structure which included many terms such as : the cost of concrete , the reinforcing steel , fiber , prestressing steel , form work , shear steeletc.

This paper deals with minimizing the steel reinforcement cost and the concrete cost of atypical one way slab using the lagrangian multipliers method .

Olhoof [3] determined the thickness of a simply supported rectangular plate , in which the fundamental frequency of transverse vibration is an optimal value , Khot , et al. [4] presented a method based on the optimality criteria for designing minimum weight fiber reinforced structure with stress displacement constraints.

Minimization of weight of a stiffened conical and cylindrical shells was carried out by Rao and reddy [5] considering practical constraints including natural frequency , they minimized the weight by appropriately selecting the shell thickness and spacing of rings and stringers using the interior penalty function method .

Brown [6] presented an iterative method for minimum cost selection of the thickness of simply supported uniformly – loaded one way slab using only the flexural constraints of the ACI – Code (" Building " 1971) , the cost function includes only the cost of concrete and the cost of reinforcing steel . The author reports cost saving of up to 17 % .

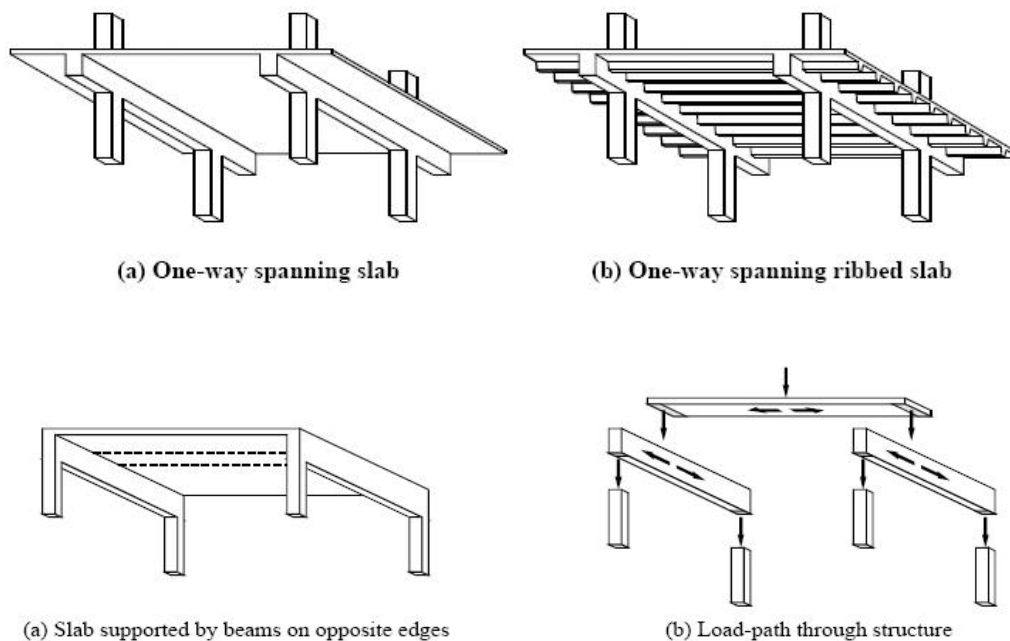


Fig. (1) Load-path in a one-way spanning slab.

Chou [7] uses the lagrange multiplayer method for minimum cost design of a simply reinforced T – Beam using the ACI Building Code – 1971 , the writer defines only two design variables : effective depth and area of steel reinforcement . The cost function includes the cost of concrete and the cost of reinforcing steel , in the formulation , its assumed that the neutral axis is located inside the flange of the T – section , the author reports a cost reduction up to 14 % of the cost of the beam with maximum steel ratio .

Gunaratnam and Sirakumaran (1978) [8] presented minimum cost design of reinforced concrete slab satisfying the limit states requirements of the British Code - 1972 for members having uniform , triangular or parabolic moment distribution using a combination of the lagrange multiplier and graphical methods . Their cost function includes only the cost of concrete and the cost of reinforcing steel . They present curve for optimum design parameters as a function of the thickness of the slab . They point out the significant influence of the serviceability limit state of deflection on the optimum design parameters .

Mathematical Formulation

Objective Function

The objective function can be expressed as :-

$$\text{Minimize } f(x) = C_t \quad (1)$$

$$C_t = V_s * C_s + V_c * C_c \quad (2)$$

Where :-

C_t : The total material cost .

C_s : The steel reinforcement cost / unit volume .

C_c : The concrete cost / unit volume .

V_s : Volume of steel .

V_c : Volume of concrete .

Equation (2) can be rewritten as :-

$$C_t = C_c (V_s * r + V_c) \quad (3)$$

Where :-

r : is the cost ratio of the cost of a unit volume of steel to a unit volume of concrete (C_s / C_c).

$$V_s = 1 * 1 * A_s$$

$$V_c = 1 * 1 * h$$

h : Total depth of the section .

ρ : Reinforcement ratio

Where :-

$$A_s = \rho * b * h$$

$$C_t = C_c (\rho * b * h * r + h) \quad (4)$$

Since $b =$ unit width (1 m) equation (4) can be

$$C_t = C_c * h * (\rho * r + 1) \quad (5)$$

Constraint Functions

The constraint functions of this problem can be explained as :-

$$g_i (x) = 0, I = 1, 2, \dots, m \quad (6)$$

Where :-

x : is the vector of constraint design variables including flexural constraints , serviceability constraints and deflection constraints .

$$M_u \leq \phi * M_n, \phi = 0.9 \quad (7)$$

Where :-

M_u : The ultimate design moment .

M_n : The nominal bending moment .

The ultimate design moment is calculated from the external loads as follow :-

$$M_u = k * w * L^2 \quad (8)$$

Where :-

k : moment coefficient .

w : The factored uniformly distributed load .

L : The effective span .

The nominal bending moment is calculated as follow :-

$$M_n = A_s * f_y (h - d^- - a / 2) \quad (9)$$

Where :-

A_s : The steel reinforcement .

f_y : Yield stress of steel reinforcement .

d⁻ : Concrete cover .

The serviceability constraints are presented in terms of limits on the steel reinforcement ratio and the bar spacing .

Where :-

$$\rho = A_s / b * h \quad (10)$$

And should satisfy the following constraints :-

$$\rho_{\min} < \rho < \rho_{\max} (0.85 * \beta_1 * (f_c^- / f_y) * (\epsilon_u / \epsilon_u + 0.004))$$

Where :-

ρ_{\min} : minimum reinforcement ratio which is equal to (0.002) for slabs where grade 280 MPa to 350 MPa , or equal to (0.0018) for slabs where grade 420 MPa , and equal to (0.0018 * 420 / f_y) for slabs where yield stress exceeding 420 MPa , but not less than 0.0014 .

$\epsilon_u = 0.003$.

$\beta 1$: is equal to (0.85) for concrete strength up to 28 MPa , and for concrete strength greater than 28 MPa $\beta 1$ shall be reduced at a rate of (0.05) for each 6.9 MPa but not less than (0.65) .

Constraint normalization [9]

$$\frac{k * w * L^2}{0.9(A_s * f_y * ((h - d^-) - \frac{(A_s * f_y / 0.85 * f_c^- * b)}{2}))} - 1 \leq 0 \quad (11)$$

$$1 - \frac{A_s}{A_{s_{\min}}} \leq 0 \quad (12)$$

$$1 - \frac{h - d^-}{(h - d^-)_{\min}} \leq 0 \quad (13)$$

By using the lagrangian multiplication method to find the optimum solution for the cost of steel and concrete for the given variables (d , ρ) , and using equations (5) and (11) , The Total objective cost function will be :-

$$C_t = C_c * h * (\rho * r + 1) - \lambda \left[\frac{k * w * L^2}{0.9 * (A_s * f_y * ((h - d^-) - \frac{(A_s * f_y / 0.85 * f_c^- * b)}{2}))} - 1 \right] \quad (14)$$

In which (λ) is the lagrangian multiplication constant that will be found during the solution with the other independent variables (d , ρ) .

Solving equation (14) by derivation with respect to the independent variables (h , ρ , λ) and equaling the resulting equations to (0) gives these three equations :-

$$\frac{\partial C_t}{\partial \lambda} = -M_o + 0.9 * \rho * h^2 * f_y - \frac{k1}{2} * \rho^2 * h^2 = 0 \quad (15)$$

$$\frac{\partial C_t}{\partial h} = C_c * (\rho * r + 1) + 1.8 * \lambda * \rho * f_y * h - k1 * \rho^2 * \lambda * h = 0 \quad (16)$$

$$\frac{\partial C_t}{\partial \rho} = C_c * (h - d^-) * r + 0.9 * \lambda * f_y * h^2 - k1 * \rho * h^2 * \lambda = 0 \quad (17)$$

Where :-

M_o : $k * w * L^2$.

$k1$: $(1.8 * f_y^2) / (1.7 * f_c^-)$

then finding the there values (h , ρ , λ) that represent the optimum in these equations :-

$$h_{opt} = 0.35136 * \frac{(9 * fy * r + 10 * k1) * \sqrt{Mo}}{fy * (9 * fy * r + 5 * k1)^{0.5}} \quad (18)$$

$$\rho_{opt} = \frac{9 * fy}{(9 * fy * r + 10 * k1)} \quad (19)$$

$$\lambda_{opt} = 0.35136 * \frac{(9 * fy * r + 5 * k1)^{0.5} * C_c}{fy * \sqrt{Mo}} \quad (20)$$

By applying the other constraints in equations (12) and (13) , ρ_{opt} and h_{opt} can be expressed as :-

$$\begin{aligned} \rho_{opt} &= \rho_{opt} && \text{If } \rho_{min} \leq \rho_{opt} \leq \rho_{max} \\ \rho_{opt} &= \rho_{min} && \text{If } \rho_{opt} \leq \rho_{min} \\ \rho_{opt} &= \rho_{max} && \text{If } \rho_{opt} \geq \rho_{max} \end{aligned} \quad (21)$$

And :-

$$\begin{aligned} h_{opt} &= h_{opt} && \text{If } \rho_{opt} = \rho_{opt} \\ h_{opt} &= h_{min} && \text{If } \rho_{opt} = \rho_{max} \\ h_{opt} &= h_{max} && \text{If } \rho_{opt} = \rho_{min} \end{aligned} \quad (22)$$

And so the values of h_{min} & h_{max} will be :-

$$h_{min} = \sqrt{\frac{Mo}{(0.9 * fy * \rho_{max} - \frac{k1}{2} * \rho_{max}^2)}} \quad (23)$$

$$h_{max} = \sqrt{\frac{Mo}{(0.9 * fy * \rho_{min} - \frac{k1}{2} * \rho_{min}^2)}} \quad (24)$$

Numerical Examples

Three examples were solved using these equations and a visual basic computer program (As seen in appendix A) developed for finding the results of these examples . First a one way slab with moment equal to 50 kN.m/m , $f_c = 20$ MPa , $f_y = 276$ MPa and $r = 75$, were solved to find the optimum depth and the optimum reinforcement ration that gives the optimum cost for the constant example comparing to the costs that results from changing the depth of the slab or the reinforcement ratio .

As it seen from Table 1 the depth of (0.142 m) and the reinforcing ration of (0.01096) gives the optimum cost solution even when the depth of the slab is less than the optimum depth that found and Fig. 2 shows the visible region [2] for this example for the

optimum cost solution , from this fig. it can be notice that the optimum solution that have been found is lying between the two point from using equations (23) and (24) .

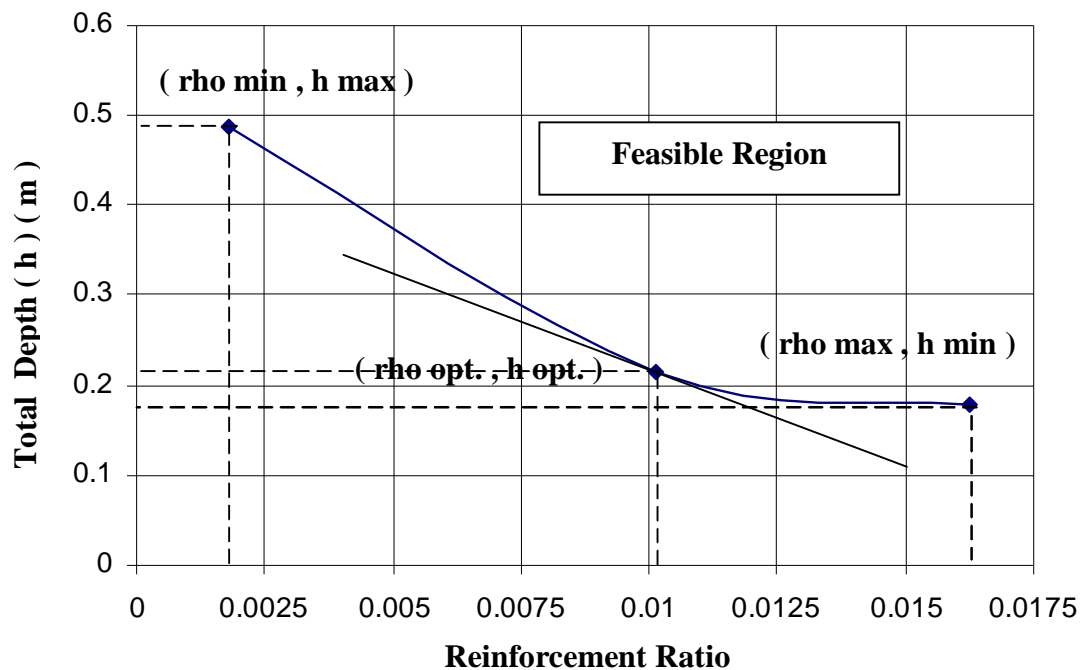


Fig (2) Feasible Region of minimum cost design for a given example of :
 $\mu = 50 \text{ kN} \cdot \text{m} / \text{m}$, $f_c = 20 \text{ MPa}$, $f_y = 276 \text{ MPa}$, $r = 75$

Table (1) Cost design for optimum solution using the lagrangian multiplier for a one way slab with : $\mu = 50 \text{ kN} \cdot \text{m} / \text{m}$, $r = 75$, $f_y = 276 \text{ MPa}$, $f_c = 20 \text{ MPa}$.

Reinforcement Ratio (rho)	Effective Depth (h) (m)	Area of Steel (mm^2)	Material Cost * Cc (\$ / m)
0.002	0.32	640	0.368
0.004	0.228	912	0.2964
0.006	0.1878	1127	0.2723
0.008	0.164	1312	0.2624
0.01096	0.142	1557	0.2587
0.014	0.127	1778	0.26
0.018	0.1144	2060	0.2688
0.0269	0.0978	2631	0.295

Figs. 3 , 4 and 5 represent the optimum cost design for three examples of $f_y = 276$ MPa , $f_y = 345$ MPa and $f_y = 400$ MPa with different values of f_c' for the optimum solution and the ACI Code 2005 solution [10] , it can be seen from these figs. that the optimum cost design is increase with the values of f_c' for the same value of f_y and also the different between the two costs (Optimum & ACI) getting larger as the value of f_y increase from (276 MPa) to (400 MPa) through the three figs.

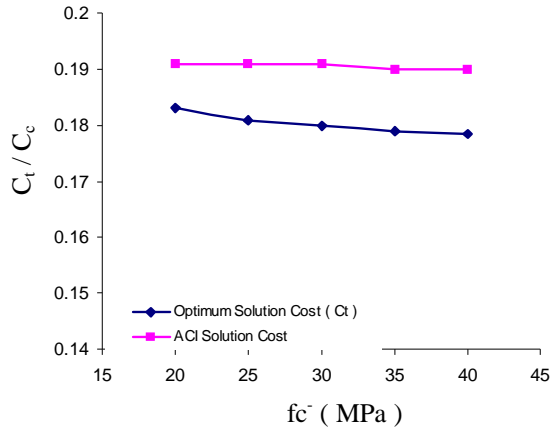


Fig. (3) : Cost design for both : optimum solution and ACI – Code solution , for a certain example with , $M_u = 25$ kN . m / , $r = 75$ and $f_y = 276$ with different values of f_c'

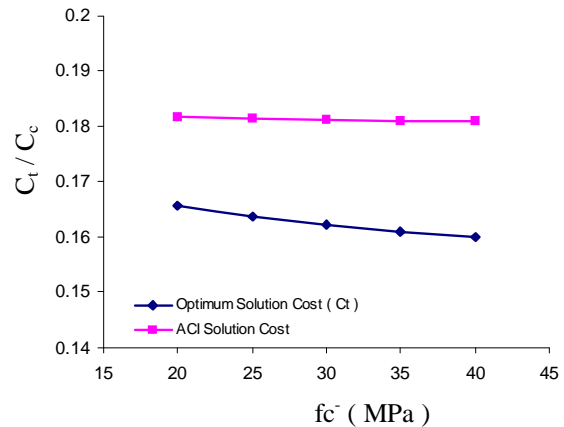


Fig. (4) : Cost design for both : optimum solution and ACI – Code solution , for a certain example with , $M_u = 25$ kN . m / , $r = 75$ and $f_y = 345$ with different values of f_c'

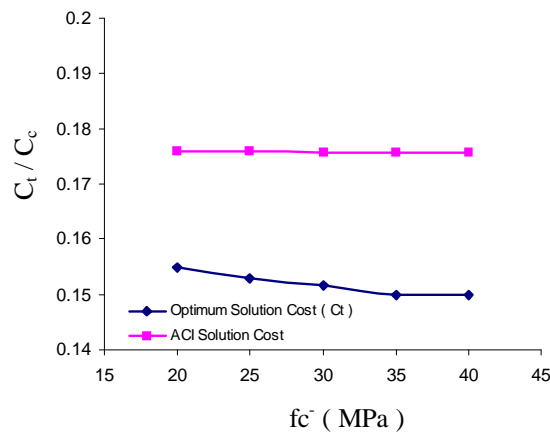


Fig. (5) : Cost design for both : optimum solution and ACI – Code solution , for a certain example with , $M_u = 25$ kN . m / , $r = 75$ and $f_y = 400$ with different values of f_c'

Tables 2 , 3 and 4 shows the minimum cost design as a factored of C_c as compare with the cost design of several examples with moment equal to (25.0 kN . m / m) , (30.4 kN . m / m) and (33.92 kN . m / m) , $r = 75$ and a different values of f_c' and f_y , the difference in the cost for these examples (Saving cost) were drawn in Fig. 6 and 7 , obviously from these figs. The minimum savings will be when using a minimum values of f_c' and f_y while using a larger values for these variables during the design gives use a better chance to be closer to save more cost .

Table (2) : Cost design for both : optimum solution and ACI – Code solution , for a certain example with , $\mu = 25.0 \text{ kN} \cdot \text{m} / \text{m}$, $r = 75$ and different values of (f_c & f_y)

Fy(Mpa)	276		345		400	
Fc(Mpa)	Opt.	ACI-05	Opt.	ACI-05	Opt.	ACI-05
20	0.183	0.191	0.1656	0.1818	0.155	0.176
25	0.181	0.191	0.1636	0.1814	0.153	0.176
30	0.1799	0.191	0.1623	0.18125	0.1517	0.1758
35	0.179	0.190	0.161	0.1811	0.150	0.1757
40	0.1784	0.190	0.160	0.181	0.1498	0.1756

Table (3) : Cost design for both : optimum solution and ACI – Code solution , for a certain example with , $\mu = 30.4 \text{ kN} \cdot \text{m} / \text{m}$, $r = 75$ and different values of (f_c & f_y)

Fy(Mpa)	276		345		400	
Fc(Mpa)	Opt.	ACI-05	Opt.	ACI-05	Opt.	ACI-05
20	0.2017	0.223	0.1826	0.2132	0.1712	0.2078
25	0.1997	0.2227	0.18	0.213	0.1688	0.2076
30	0.1984	0.2224	0.1789	0.2127	0.167	0.2074
35	0.1974	0.2223	0.1779	0.2126	0.1661	0.2073
40	0.1967	0.2222	0.177	0.2125	0.165	0.2072

Table (4) : Cost design for both : optimum solution and ACI – Code solution , for a certain example with , $\mu = 33.92 \text{ kN} \cdot \text{m} / \text{m}$, $r = 75$ and different values of (f_c & f_y)

Fy(Mpa)	276		345		400	
Fc(Mpa)	Opt.	ACI-05	Opt.	ACI-05	Opt.	ACI-05
20	0.213	0.23	0.193	0.218	0.181	0.212
25	0.211	0.228	0.19	0.2176	0.178	0.2116
30	0.2096	0.2282	0.189	0.217	0.1767	0.2114
35	0.208	0.228	0.1879	0.217	0.175	0.211
40	0.2078	0.2278	0.187	0.217	0.1746	0.211

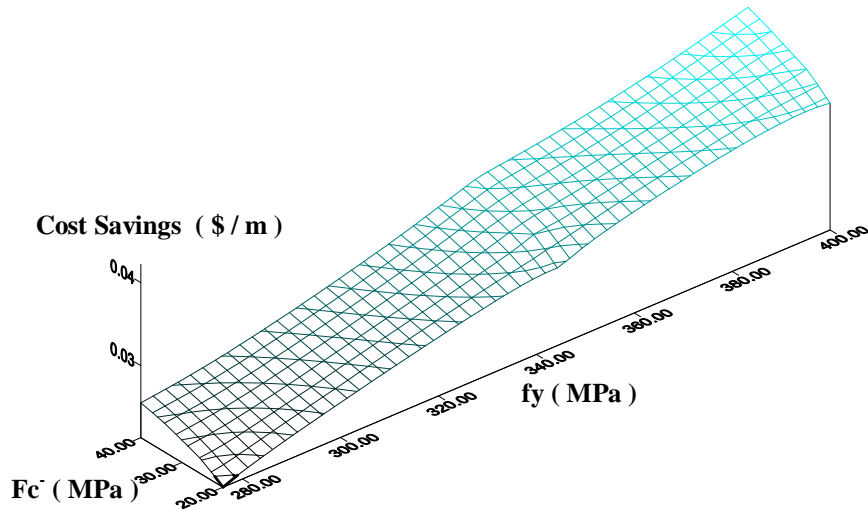


Fig. (6) : Cost saving between optimum solution and ACI – Code solution , for a certain example with , $\mu = 30.4 \text{ kN} \cdot \text{m} /$, $r = 75$ and different values of (f_c' & f_y)

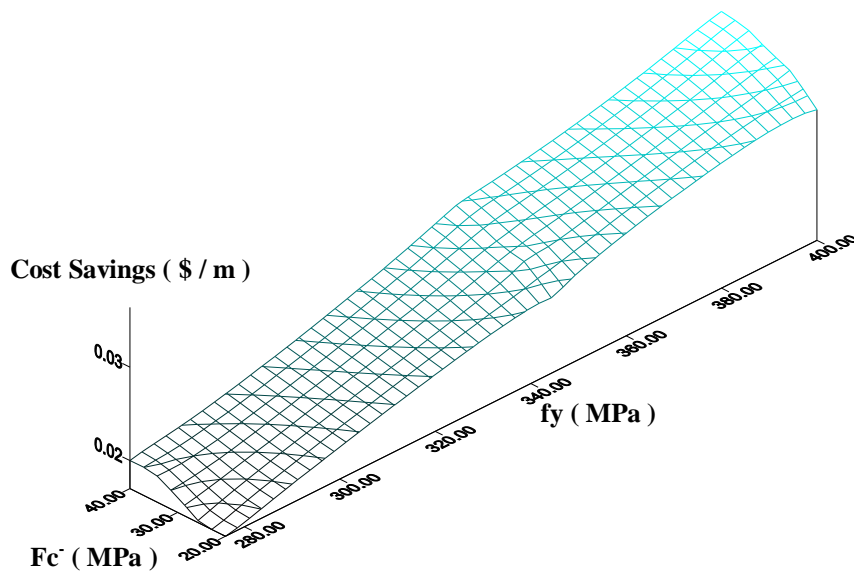


Fig. (7) : Cost saving between optimum solution and ACI – Code solution , for a certain example with , $\mu = 33.92 \text{ kN} \cdot \text{m} /$, $r = 75$ and different values of (f_c' & f_y)

Conclusion

Optimum cost have been found using the lagrangian multiplication method for the reinforcing steel ratio and the effective depth for a Reinforced concrete one way slabs , and the following conclusions have been found :-

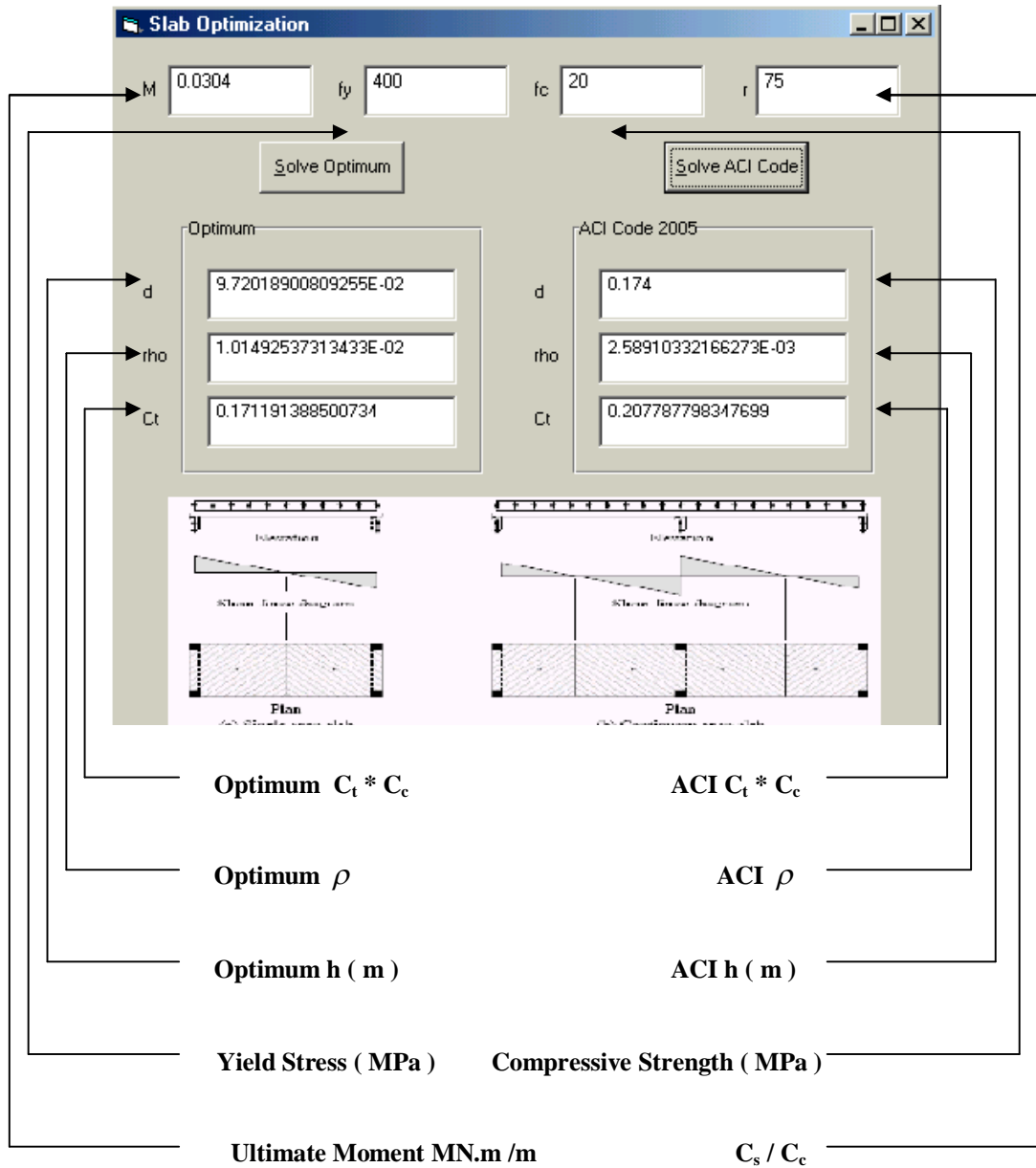
1 – The equations driven based on eq. 18 and eq. 19 represent the optimum cost solution since it gives the lower cost for any one way slab even when the depth is less than the one that found from the optimum depth equation .

- 2 – A saving cost up to 20 % , have been found in some of the presented examples using this method .
- 3 – The savings in the cost is more noticeable for the slabs having higher values for the yield stress and the concrete compressive strength .
- 4 – Its more effecting to have other constraints such as frame work constraints and Labors Constraints ... etc.
- 5- The produced visual basic program in appendix A can be developed to solve many other problems such as tow way slabs cost optimization or single footing optimization with different constraints functions .

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Appendix (A)



The work was carried out at the college of Engg. University of Mosul